Connections between special relativity, charge conservation, and quantum mechanics

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Abstract

Examination of the Einstein energy-momentum relationship suggests that simple unbound forms of matter exist in a four-dimensional Euclidean space. Position, momentum, velocity, and other vector quantities can be expressed as Euclidean four-vectors, with the magnitude of the velocity vector having a constant value, the speed of light. We see that charge may be simply a manifestation of momentum in the new fourth direction, which implies that charge conservation is a form of momentum conservation. The constancy of speed implies that all elementary free particles can be described in the same manner as photons, by means of a wave equation. The resulting wave mechanics (with a few small assumptions) is simply the traditional form of quantum mechanics. If one begins by assuming the wave nature of matter, it is shown that special relativistic results follow simply. Thus we see evidence of a strong connection between relativity and quantum mechanics. Comparisons between the theory presented here and Kaluza-Klein theories reveal some similarities, but also many significant differences between them.

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I. INTRODUCTION

One of the most important results of twentieth century physics is Einstein's theory of special relativity. Since its introduction in 1905 [1], our view of the physical world has been forever changed. We now have a much better understanding of the relationship between matter and energy than we ever had before relativity. No physics education is complete nowadays without a study of special relativity.

Of course, such studies are seldom painless. Although relativity follows from some simple principles, the logic we use to derive its major results can be quite complicated. Material objects which possess rest mass must be treated differently than those that do not in many ways. For example, the derivation of the transformation for the energy of a particle with rest mass may involve a lengthy discussion of two-body collisions, as can be found in Jackson's section 11.5 [2]. The same derivation for a photon is somewhat simpler, involving only a derivation of the relativistic Doppler shift and the quantum assumption $E = \hbar\omega$, as can be seen in section 11.2 of [2]. Although we seek the transformation for the same quantity, we find we must use two different methods to find two different results. It would certainly be desirable if we could treat both types of matter the same way. It should be noted that there are some strong similarities between the two formulae. For example, the formula for the energy of a resting body boosted to a speed v is the same as that of a photon experiencing a transverse boost of the same magnitude. It would appear that this is a mere coincidence. Is it?

In this paper we shall see that it is not mere coincidence. It is possible to add one dimension to our three dimensions of space in such a way that we can treat simple objects with and without rest mass exactly the same way. This new four-space has the nice property of being Euclidean, and yields all of the usual relativistic properties through a few simple, familiar postulates. By considering a few well-understood physical processes, we shall see that the momentum of a particle in the fourth direction may correspond to its charge, which means that charge conservation is just a form of momentum conservation. We shall also see

how easy it is to infer (with a few small assumptions) that wave and quantum mechanics offer an excellent description of matter in this four-space. In fact, we can start with four-dimensional quantum mechanics and derive the results of special relativity very naturally, with even fewer assumptions. The connection between relativity and quantum mechanics appears to be stronger than anyone previously indicated.

Before continuing, let us first define the terminology that we shall use in this paper. It is standard practice in physics today to refer to the mass of a resting classical object simply as mass. Here we will instead use the term rest mass for that quantity, and reserve the term mass for an object's total relativistic mass. This convention is commonly used in many elementary treatises on relativity (such as [4] and [6]), and fits in well with the rest mass–relativistic mass notation found in beginning physics textbooks like [3] and [5]. So here mass is related to total energy by the famous relationship $E = mc^2$. At the very least, this choice of terminology should cause no ambiguity, since most readers have probably encountered it before. Hopefully it will make the discussion more clear as well.

II. EXTENSION OF SPACE TO FOUR DIMENSIONS

Let us first examine a well-known result of special relativity. For any particle of matter,

$$E^2 = p^2c^2 + m_0^2c^4. \tag{1}$$

where E, p and m_0 are the particle's energy, momentum, and rest mass, respectively, and c is the speed of light. For reasons which shall become clear later, we restrict this discussion to extremely simple particles, such as electrons. Now let us make the following substitutions, also from special relativity:

$$m_0 = m\sqrt{1 - \frac{v^2}{c^2}} \tag{2}$$

and

$$\boldsymbol{p} = m\boldsymbol{v}. \tag{3}$$

We find that

$$E^2 = m^2c^2[v^2 + (c^2 - v^2)] \tag{4}$$

Rather than make the obvious simplification to the bracketed term in (4), let us first examine each of its parts separately. If the particle has the usual Cartesian three-space coordinates x, y, and z, then

$$v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2. \tag{5}$$

We can make this substitution for the first term in the parentheses of Eq. (4), but what about the second term? We can find an expression for it by considering "proper time" t_0. For an observer in a given reference frame, the amount of time t that passes while a time interval t_0 passes in a frame moving at speed v (the proper time) is given by the equation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{6}$$

which implies that

$$t_0 = t\sqrt{1 - \frac{v^2}{c^2}}. \tag{7}$$

Simple calculus shows that

$$\frac{dt_0}{dt} = \sqrt{1 - \frac{v^2}{c^2}}, \tag{8}$$

so

$$c^2\left(\frac{dt_0}{dt}\right)^2 = c^2 - v^2. \tag{9}$$

We can now express (4) as

$$E^2 = m^2c^2\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 + c^2\left(\frac{dt_0}{dt}\right)^2\right]. \tag{10}$$

Now we may complete the obvious sum in (4).

$$E^2 = m^2c^4 \tag{11}$$

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Solving (10) and (11) simultaneously brings us to an important result,

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 + c^2\left(\frac{dt_0}{dt}\right)^2 = c^2. \qquad (12)$$

Let us now introduce a new quantity, w, such that $\frac{dw}{dt} = \pm c\frac{dt_0}{dt}$. (We shall see the reason for the sign ambiguity in section III.) Eq. (12) becomes

$$\left(\frac{dw}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = c^2. \qquad (13)$$

We have expressed (13) in the form of an ordinary self inner product for a vector in a Euclidean four-space. The last three terms of the sum were collectively known as $\boldsymbol{v} \cdot \boldsymbol{v}$; now we have an extra term added. We can incorporate the extra term by simply adding another, orthogonal direction for $\frac{dw}{dt}$ to our vector space. Let us call that direction $\hat{\boldsymbol{h}}$, and the directions for $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dz}{dt}$ will be the traditional $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ (respectively) of most elementary physics textbooks. In other words, a simple particle of matter can be described as traveling through a Euclidean four-space whose position vector $\boldsymbol{r}$ can be described as

$$\boldsymbol{r} = w\hat{\boldsymbol{h}} + x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}. \qquad (14)$$

Its velocity vector $\frac{d\boldsymbol{r}}{dt}$ is constrained by the equation $|\frac{d\boldsymbol{r}}{dt}| = c$. If we consider instead a photon, whose three-space speed is c, the quantity $\frac{dw}{dt} = 0$, so we can use the same four-space described above. This four-space is sufficiently general to describe all kinds of matter.

In summary, simple particles of matter can be described as existing in a Euclidean four-space as described above, with the proper time of the particle being related to the fourth, non-obvious position component. In such a space, all simple forms of matter are constantly moving at the speed of light. Later we shall see how more complicated forms behave.

III. PHYSICAL INTERPRETATIONS

As we saw earlier, $\frac{dw}{dt}$ is related to the rate of change of proper time for a simple particle, so $|w|$ is, in a classical sense, a measure of the age of a simple particle. Notice that in Eq.

(13), we must allow $\frac{dw}{dt}$ to be either positive or negative. What is the physical meaning of such a sign?

To answer that question, let us return again to (10), slightly modified:

$$E^2 = m^2 c^2 \left[\left(\frac{dw}{dt} \right)^2 + \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2 \right]. \tag{15}$$

The bracketed quantity again appears to be a self inner product of something fundamental. Let $\boldsymbol{u} \equiv \frac{d\boldsymbol{r}}{dt}$ be our particle's velocity in our new four-space; (15) becomes

$$E^2 = (m\boldsymbol{u})^2 c^2. \tag{16}$$

Let us now define the momentum $\boldsymbol{p}$ of a particle in the usual way, $\boldsymbol{p} = m\boldsymbol{u}$, so

$$E^2 = \boldsymbol{p}^2 c^2. \tag{17}$$

Thus all simple matter can be described by the free photon energy-momentum relationship $E = pc$. As with the photon, we presume that E is always positive, which makes perfect sense because E is merely a measure of the magnitude of a particle's momentum. Since $E = mc^2$, we should make the same assumption for m.

Now we have a four-dimensional Euclidian momentum space in addition to the four-dimensional position space. The momentum space is defined as before, with the added component

$$\boldsymbol{p}_w = m \left(\frac{dw}{dt} \right) \hat{\boldsymbol{h}} = \pm m_0 c \hat{\boldsymbol{h}}. \tag{18}$$

The magnitude of the extra momentum component in these simple particles is proportional to their rest mass. Notice the qualifier "simple." Can one use (18) for *any* particle?

Momentum conservation is one of the most useful concepts in physics, with countless practical applications and experimental verifications. One would expect that if rest mass is a form of momentum, then it must conserve in a collision. There are, however, many examples of physical processes that disprove that idea. For example, rest mass does not conserve in an electron capture process, where a proton and an electron combine to form a

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neutron. Thus (18) cannot be true for all particles. It does appear to be correct for simple particles, however. Can we find another physical quantity that agrees for all particles? We can, and we will now do so by examining some very simple and well-understood particle interactions.

Let's start with pair annihilation, in which an electron and a positron are converted to photons. Since neither the electron nor the positron can be broken down into smaller components, it seems reasonable to assume that they are among the simplest forms of matter, and that they obey Eq. (18). We expect momentum to conserve here. Since the outgoing photons are traveling at light speed in three-space, they must each have $p_w = 0$. To ensure momentum conservation, the electron and the positron must have w-momenta equal in magnitude but opposite in sign. There are a few differences between the electron and the positron, but the fact that they have opposite charges is the most obvious and important one. It seems very reasonable to assume that the sign of the charge of a particle is related to the sign of its w-momentum component, and that charge and w-momentum are somehow equivalent.

We can find supporting evidence for this idea by examining the results of some more complicated interactions, which are all summarized in [7]. Let us assume that an electron has $p_w = -m_e c$ (where m_e is its rest mass), and a positron has $p_w = m_e c$. A negatively charged muon is known to decay into an electron and a pair of neutrinos. Neutrinos appear to lack rest mass [7], so they must also lack w-momentum. We expect w-momentum to conserve, so the electron and the muon must have the same w-momentum. Since the negative tauon can also decay to an electron and two neutrinos, it must also have $p_w = -m_e c$. A negative tauon can also decay to a negative pion and a neutrino, so the negative pion must also have $p_w = -m_e c$. We can conclude that electrons and negative muons, tauons, and pions all have the same w-momentum. It appears that the w-momentum of a particle depends only on its charge q. We can express this idea mathematically as

$$p_w = -\frac{q m_e c}{e},$$

(19)

where e is the charge of an electron. It is well-known that charge conserves; we can now see that charge conservation may be simply another form of momentum conservation.

IV. WAVE MECHANICS AND QUANTUM MECHANICS

Earlier we observed that, with the inclusion of a form of proper time as a spatial coordinate, simple free particles can be viewed as traveling at the speed of light at all times. This property of constant velocity strongly suggests that simple free particles can be described mathematically in the same manner as photons; they are simply solutions to some form of the wave equation. In this section we shall explore this idea and examine its consequences.

Let $\boldsymbol{\Psi}(\boldsymbol{x}, t)$ be the wave function for a free particle at position $\boldsymbol{x}$ and time t. The dimensionality of $\boldsymbol{\Psi}$ is not important for this discussion at this time. Also, let x_j ($j = 1, 2, 3, 4$) be a set of Cartesian coordinates that span our four-dimensional space, with $w = x_4$. Our wave equation is:

$$\sum_{j=1}^{4} \frac{\partial^2 \boldsymbol{\Psi}}{\partial x_j^2} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{\Psi}}{\partial t^2}. \tag{20}$$

Solutions to this equation are well-known. In these coordinates, we can say that the eigenfunction $\boldsymbol{\Psi}$ for wave vector $\boldsymbol{k}$ is

$$\boldsymbol{\Psi}(\boldsymbol{x}, t) = \boldsymbol{\Psi_0} e^{i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)}. \tag{21}$$

where $\omega = \frac{|\boldsymbol{k}|}{c}$. To turn this wave mechanical system into a form of quantum mechanics, we need only one extra assumption. We must simply relate the momentum and wave vectors through the equation $\boldsymbol{p} = \hbar \boldsymbol{k}$ and our wave mechanics becomes the quantum mechanics of free photons.

One useful result of quantum mechanics is the operator formalism. The forms of the operators often seem cryptic, but one nice feature of the wave mechanics we are using is that the operators follow rather intuitively. To find them, all we need to do is take our eigenfunctions and ask what we need to do to them to find the properties which we are interested in. For example, simple calculus tells us that, for a function of the form of (21),

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$$-i\hbar\frac{\partial\boldsymbol{\Psi}}{\partial x_j} = p_j\boldsymbol{\Psi}. \tag{22}$$

We could mathematically extract p_j from this equation, but we know that the operator form is useful in and of itself. We can use Eq. (22) as a definition for momentum in more complicated systems, by defining the momentum operator $\hat{P}_j$ as

$$\hat{P}_j = -i\hbar\frac{\partial}{\partial x_i}. \tag{23}$$

This result is simply the operator notation found in most quantum mechanics textbooks (for example, see [8] and [9]). The arguments that once led to these operators were traditionally quite complicated, so much so that most introductory textbooks will not discuss them, but now we have a more intuitive explanation. Using the same reasoning, one can also define the quantum mechanical energy operator $\hat{E}$ as

$$\hat{E} = i\hbar\frac{\partial}{\partial t}. \tag{24}$$

By simply multiplying (20) by $-\hbar^2 c^2$ we see that the expected relation

$$\hat{E}^2\boldsymbol{\Psi} = \hat{P}^2 c^2\boldsymbol{\Psi} \tag{25}$$

holds true.

Equation (25) appears to be a quantum mechanical equation for free photons, assuming that $\boldsymbol{\Psi}$ is a scalar, Ψ. How can we turn this into a corresponding relationship for free particles with rest mass? Let us start by substituting in a form of (18), namely

$$p_4\Psi = -i\hbar\frac{\partial\Psi}{\partial x_4} = \pm m_0 c\Psi \tag{26}$$

and (25) becomes

$$-\hbar^2\frac{\partial^2\Psi}{\partial t^2} = -\hbar^2 c^2\nabla^2\Psi + (m_0 c^2)^2\Psi. \tag{27}$$

Rearranging and dividing by $\hbar^2 c^2$ gives

$$-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} + \nabla^2\Psi = \left(\frac{m_0 c}{\hbar}\right)^2\Psi, \tag{28}$$

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which is of course the Klein-Gordon equation. It is now a simple matter to reduce (28) to the free particle Schrödinger equation.

Thus we can infer from some simple relativistic considerations that quantum mechanics is applicable in our four-space. With only a few fairly intuitive assumptions, one can derive for free particles the quantum mechanical wave equation, the Klein-Gordon equation, and the Schrödinger equation. Without the addition of a fourth orthogonal dimension to our definition of space, quantum mechanics and relativity are independent theories. In our Cartesian four-space, however, we can see that there is in fact a strong interdependence between them.

V. REST MASS IN PHYSICAL SYSTEMS

Earlier we established that the fourth momentum component of a particle was not necessarily related to its rest mass. In this section we shall take a closer look at rest mass, and find out what it is and why relates to mass in the standard relativistic sense. We shall use quantum mechanics as the tool for our investigation.

Imagine two free particles overlapping in space in the center-of-mass reference frame. Let them have wave vectors $-\boldsymbol{k}$ and $\boldsymbol{k}$ respectively. Their wavefunctions are

$$\boldsymbol{\Psi_1}(\boldsymbol{x}, t) = \boldsymbol{\Psi_0} e^{i(-\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)} \tag{29}$$

and

$$\boldsymbol{\Psi_2}(\boldsymbol{x}, t) = \boldsymbol{\Psi_0} e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}. \tag{30}$$

From the operators defined in section IV we can see that the momentum for the combination must be 0, but the energy E is equal to $2\hbar|\boldsymbol{k}|$. The combination is in a zero momentum state, but it has a significant amount of energy. If we use the standard relativistic definition of mass ($E = mc^2$), we can say that the rest mass of this system is $\frac{2\hbar|\boldsymbol{k}|}{c^2}$.

So how does the energy of this system transform after a velocity shift $\boldsymbol{v}$? We can think of the two wave equations as being the same as those for photons with opposite momenta.

The formula for the energy transformation can be determined from adding Doppler shift results for each wave function, which are

$$E' = \frac{E - \boldsymbol{v} \cdot \hbar \boldsymbol{k}}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{31}$$

Because we are starting with two waves of opposing momenta, the final result, independent of the direction of the velocity shift, is

$$E' = \frac{E}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{32}$$

so in the transformed reference frame, if E' is the transformed energy and E is the untransformed energy, we can use the relationships $E = \hbar\omega$ and $E = mc^2$ to find that

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{33}$$

where m_0 is a constant. It can now be easily shown that for this multiparticle system, mass obeys Eq. (1), regardless of the direction of the velocity change. It is interesting to notice that there is no w-momentum in this system, but there is rest mass. Rest mass is not necessarily, then, a phenomenon of momentum in systems composed of several particles.

For systems composed of more than two particles, the same results apply, which can be seen by simply pairing off particles with opposing momenta. If an excess of w-momentum exists in the system, we will not notice any strange effects, because all of our velocity shifts are done in three-space. Rest mass and w-momentum are both invariant.

We can now see that $\boldsymbol{p}$ is a four-vector, but in many cases of interest, one can simply use the three-vector form instead. For example, it is currently common practice to simply assume that electrons and positrons have rest mass instead of w-momentum; this approximation works well in many circumstances. For compound systems which are neutral, there is no w-momentum, so the three-vector solution is exact. For charged compound systems, since we do all of our velocity shifts in three-space, rest mass and w-momentum are both invariant. One could treat $p_w{}^2c^2 + m_0{}^2c^4$ in Eq. (1) as a single rest mass term; there are no differences at this time.

VI. DERIVATIONS OF SPECIAL RELATIVISTIC PHENOMENA

Let us now look back at our four-dimensional quantum mechanics. When we derived its properties in section IV, we started from relativistic considerations and developed quantum mechanics after making a few assumptions. Here we shall see that if we start from a four-dimensional quantum mechanical perspective, we can derive the key results from special relativity very easily.

As a starting point, we need an expression for quantum mechanics. Since special relativity is a theory for free particles, let us assume that there are no interactions. Our quantum mechanical wave equation is simply equation (20), with the eigenfunctions as shown in equation (21).

We should first derive the Lorentz transformations. A common way to derive them involves the *gedankenexperiment* of examining the travel of a burst of light propagating in all directions from the perspective of two different reference frames, as Einstein first did [1]. We can derive these transformations in exactly the same way in our four-dimensional space. There is an added direction, which turns Einsteins three-dimensional sphere of light into a four-dimensional sphere described by the equation

$$w^2 + x^2 + y^2 + z^2 = c^2 t^2 \tag{34}$$

but there are no other differences in the proof. The w-direction is simply another direction orthogonal to a special relativistic velocity shift, so the transformation of w is simple, $w' = w$. So distance in the w-direction is invariant in special relativity. This is not new information, however. Let us rewrite Eq. (34) in a differential form ($x \rightarrow dx$, etc.) and solve for dw^2:

$$dw^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{35}$$

We can now see dw for what it truly is; it is the definition of what is commonly referred to as the spacetime interval, which was first discussed by Minkowski [10]. The spacetime interval is simply another distance for a direction orthogonal to the direction of the velocity shift, and now has a fairly simple explanation.

Now that we have the Lorentz transformations, the energy and momentum transformations follow in a simple manner. A free particle is described as a combination of eigenfunctions of the form of Eq. (21). We can examine each eigenfunction individually. Observers in all reference frames must agree on the phase of an eigenfunction, for exactly the same reason that observers in different frames must agree on the phase of a photon's wavefunction. In other words, by adding an extra dimension to space, we can treat photons and particles with rest mass in exactly the same way. For photons, we can derive the relativistic Doppler shift equations to find the energy and momentum transformations (see section 11.2 of [2]); these same formulae apply to other kinds of matter as well in a quantum mechanical four-space. There is absolutely no difference in the way these types of matter are treated.

Thus we have shown that quantum mechanics and special relativity are very closely related. All of the properties of matter in special relativity are simple consequences of the four-dimensional quantum mechanics we are considering. It is interesting to note that relativistic properties followed very simply from quantum mechanics, whereas we needed to make several assumptions (albeit simple ones) to derive our quantum mechanics from relativistic considerations. Instead of saying that our quantum mechanics is relativistic, perhaps it would be better to say that special relativity is quantum mechanical.

VII. COMPARISON WITH KALUZA-KLEIN THEORIES

Those readers who are familiar with the general theory of relativity may see some similarities between the ideas proposed here and Kaluza-Klein theories [11]. These theories in their basic form have a number of similarities to the idea proposed above, such as the addition of another dimension of space to explain electric charge. In this section we shall discuss the similarities between my theory and Kaluza-Klein theories. We shall see that although they share some common characteristics, they are in fact different in many respects, enough so that they cannot be considered the same theory.

Let us begin with a very brief description of Kaluza-Klein theories. A Kaluza-Klein space

consists of a four-dimensional space as described by general relativity (three dimensions of space plus time) plus a fifth, spacelike dimension. The fifth dimension of Kaluza-Klein theories is used in a very similar way to the theory presented above, to explain the electric charge of an object. The fifth dimension can be thought of as a small circle with circumference l, which leads to quantization of electric charge.

Now let us compare the two theories. In the theory presented above, no condition of circularity was imposed on the w-dimension; space is assumed to be infinite in all dimensions, which is a large difference between the theory presented above and Kaluza-Klein theories. My theory is different in other respects as well. As we saw above, my theory has its origins in special relativity, so space is flat. Kaluza-Klein theories have their roots in general relativity, and their spaces are curved and closed in at least one dimension. Also, the two theories serve different purposes. Kaluza-Klein theories were originally created in an attempt to unite fundamental forces (such as electromagnetism and gravitation) in a general relativistic theory. My theory demonstrates a different kind of unification, between quantum mechanics and special relativity.

The different natures of these two theories leads to very different physical predictions. Although in this paper we did not make the w-dimension periodic, we could do so in order to quantize charge. There may be other explanations for charge quantization, but let us assume the w-dimension is periodic here purely for the sake of argument. One might argue that by doing so, and by assuming that the Kaluza-Klein spacetime lacks gravitation, Kaluza-Klein theory and the theory presented here are one and the same. This statement is incorrect. The circumference of Kaluza-Klein theory was calculated by Klein [14] to be

$$l = \frac{hc\sqrt{2\kappa}}{e} = 8 \times 10^{-33} \text{m}, \tag{36}$$

where κ is the Einstein gravitational constant. In contrast, the theory presented in this paper indicates that l is the wavelength of an electron with no three-space momentum. This wavelength is commonly known as the Compton wavelength, so

$$l = \frac{h}{m_e c} = 2.426 \times 10^{-12} \text{m}, \tag{37}$$

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which is many orders of magnitude larger than Klein's result.

In summary, the two theories are in fact quite different from one another, despite their similar uses of extra dimensions. The theories have different origins; Kaluza-Klein theories derive from general relativity, while my theory derives from special relativity and empirical observation. The theories attempt to unify completely different aspects of physics; Kaluza-Klein theories attempt to unify physical forces, while my theory attempts to unify different physical pictures, those of quantum mechanics and relativity. The shapes of the spaces are quite different; Kaluza-Klein theories use curved spaces closed in at least one dimension, while the theory described here uses an infinite flat space. Even if we make an effort to shape the two spaces similarly, as we did above, we find we are forced to size them differently.

VIII. CONCLUSION

By defining a fourth component in position space related to the relativistic proper time and the spacetime interval, we can describe the velocity of any elementary particle of matter as having the magnitude of the speed of light. The resulting four-space is Euclidean, which is a nice consequence in and of itself. A corresponding fourth momentum component, related to the charge and rest mass of the elementary particle, can also be defined. As a consequence, any elementary free particle can be treated exactly the same way, regardless of rest mass. Thus we can unify our description of free particles to a large extent.

There are several interesting consequences of this new four-space. For example, conservation of charge may simply be a restatement of w-momentum conservation. Also, the constant velocity condition strongly suggests that matter is obeying a wave equation, as photons do, so quantum mechanics is in fact a very natural way to describe physical phenomena in this four-space. Also, one can work backwards and show that relativistic consequences result in a simple way from using this quantum mechanical system. It took some intuition to derive quantum mechanics from relativity, but relativity is a simple consequence of quantum mechanics. One may still argue about which is more fundamental, relativity or quantum

mechanics, but this argument has no meaning. The two theories cannot be neatly separated anymore. We can now see that they are like two different facets of the same gem.

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